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# Inverted Slider-Crank Mechanism Driven by Hydraulic Cylinder: Transfer Functions and Approximations

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**Abstract:** Inverted slider-crank mechanisms driven by hydraulic cylinder have highly non-linear transfer functions, which in this form complicate kinematic and dynamic researches. A central slider-crank mechanism scheme is used with the specific small parameter equal to the ratio of the lengths of both links of the revolute pair of the mechanism ($\lambda=R/L<1$). The present study considers the two main transfer functions of the mechanism. In the first case the angle of the revolute pair as an independent parameter is accepted and in the second case the linear motion of the hydraulic cylinder as an independent parameter is accepted. The exact transfer functions of the mechanism are described and approximate representations of the transfer functions are found. In the first case we use a binomial order of the degrees of the small parameter calculated up to 4-th degree and very high accuracy of approximate function has been achieved (maximal error less than 1.6%). In the second case we use a trigonometric function, which corresponds to the exact transfer function up to second derivative, and the accuracy is also high (error less than 2%) in the main operating range. The power characteristics of the inverted slider-crank mechanism driven by hydraulic cylinder are determined using the transfer functions. All main conclusions are interpreted by geometrical representations.

**Keywords:** Inverted Slider-Crank Mechanism, Transfer Functions, Small Parameter, Approximation

## 1. Introduction

Inverted slider-crank mechanisms, realized with a revolute pair and a hydraulic cylinder as a sliding pair, are widely used in the practice. They are included in the kinematic schemes of automatic devices [1], industrial robots [2], construction machines, baggers, hydraulic escavators [3-6], etc. Unlike the slider-crank mechanism, here the sliding pair is the drive. These mechanisms are characterized by great power capabilities, simple structure and high reliability. Usually, in most cases few such mechanisms are included in the kinematic chains. For example, the kinematic chain of hydraulic escavators includes few rigid links connected with revolute pairs and driven one to another by hydraulic cylinders, connected to the links by revolute pairs [7, 8]. This way the chain with several inverted slider-crank mechanisms are formed, arranged sequentially. Thus, in order to study these complex kinematic chains, the properties of the constituent mechanism must be well known. A characteristic feature of the inverted slider-crank mechanism is its highly nonlinear transfer function between sliding and rotation. Many authors have studied this mechanism with respect to transfer functions using different methods [9-11]. The aim of this study is to describe and approximate transfer functions in a simpler form, convenient for the needs of kinematic and dynamic analysis, and with maximum approximation and minimum error.

## 2. Kinematic Scheme of the Mechanism

Figure 1 shows the mechanism under consideration. In the revolute pair AOB we denote the length of the shorter link with $R$ and the length of the longer link - with $L$. This way we define the small parameter $\lambda$, basic characteristic of this mechanism:

$$\lambda = \frac{R}{L} < 1 \quad (1)$$

We denote by $s$ the stroke of the cylinder and by $\rho = BA$ - the variable distance between the joints A and B of the cylinder. The relationship between $s$ and $\rho$ is as follows:

$$\rho = L - R + s, \ where\ 0 < s < 2R \tag{2}$$

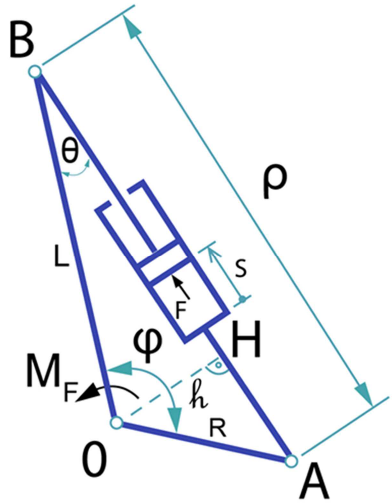


***Figure 1.*** *Inverted slider-crank mechanism.*

When studying the kinematics of the inverted slider-crank mechanism, it can be considered as an independent parameter both the angle of rotation $\varphi$ of the revolute pair AOB and the linear motion of the cylinder $s$, resp. $\rho$. We will consider both cases consequently.

# 3. Transfer Functions of Inverted Slider Crank Mechanism, with Independent Parameter - The Angle of Rotation φ

### *3.1. The Exact Transfer Function*

For an independent parameter we choose the angle $\varphi$ of the revolute pair AOB. From the closed vector contour AOB we easily obtain the following positional equations:

$$\rho \sin\theta = R\sin\varphi \tag{3}$$

$$\rho\cos\theta = L - R\cos\varphi$$

From here we obtain the expression for the linear motion of the hydraulic cylinder $\rho$:

$$\rho = L\sqrt{1+\lambda^2-2\lambda\cos\varphi} \tag{4}$$

After differentiating (4) by $\varphi$ we obtain for the first transfer function the following equation:

$$\frac{d\rho}{d\varphi} = R\frac{\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} = R.f(\varphi), \tag{5}$$

where *f(φ)* is an analogue of the dimensionless transfer function (Figure 2), known in the literature in the following form [12]:

$$f(\varphi) = \frac{\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} \tag{6}$$

### *3.2. Geometric Interpretation of the Transfer Function dρ/dφ of the Inverted Slider-Crank Mechanism*

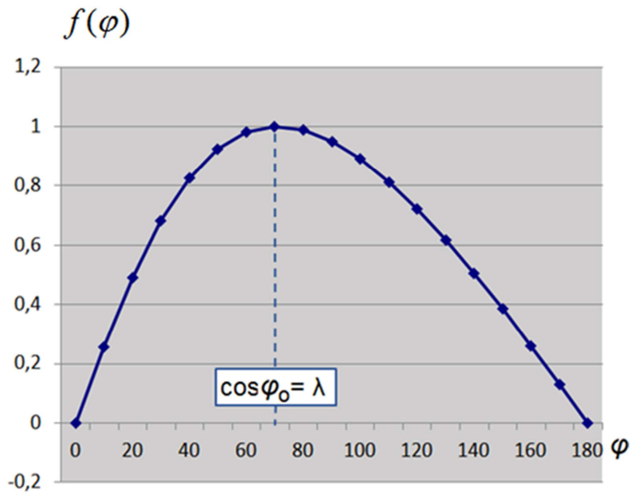


***Figure 2.*** *Transfer function f(φ) in case of λ=0.33.*

If we denote by $h = OH$ the straight line perpendicular from point O to the axis AB of the cylinder, for which it is obvious:

$$h = L\sin\theta \tag{7}$$

Substituting in (7) $\sin\theta$ expressed by (3) and $\rho$ expressed by (4), we obtain:

$$h(\varphi) = R\frac{\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} = \frac{d\rho}{d\varphi} \tag{8}$$

which corresponds to the relation evaluated in [13] as *h=L.R.sin(φ)/ρ*.

Since *h(φ)* is the perpendicular from the point O to the axis of the cylinder, on which axis acts the force of the cylinder F, then:

1. The geometric image of the transfer function *dρ/dφ* is the lever arm *h(φ)* of the cylinder force F with respect to the point O;
2. This way the moment that the force of the cylinder creates relative to the point O of the pair AOB is:

$$M_F(\varphi) = F.h(\varphi) = F\frac{d\rho}{d\varphi} = F\frac{R\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} \tag{9}$$

### *3.3. Power Capabilities of Inverted Slider-Crank Mechanism Driven by Hydraulic Cylinder*

In this mechanism, the input driving force of the linear hydraulic cylinder is transformed into the driving moment of the pair AOB on the exit. This is the driving of the rotating pairs of the open multi-link chain of these manipulators, for example the hydraulic excavators [14, 15]. Therefore, it is important to determine the force capabilities of this mechanism as a function of the angle φ of the pair AOB.

The ratio between the moment of the exit - $M_F$, and the force of the cylinder F multiplied by the constant $R$, is given by the known function *f(φ)*, which is analogous to the transfer function of inverted slider-crank mechanism in dimensionless form:

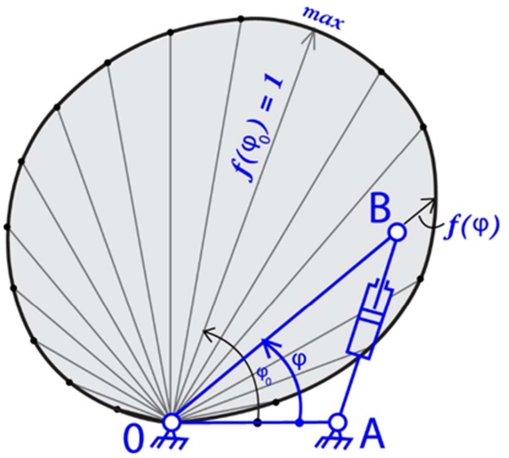


***Figure 3.*** *Shell diagram of the transfer function f(φ) in polar coordinates.*

$$\frac{M_F(\varphi)}{F.R} = \frac{\sin\varphi}{\sqrt{1+\lambda^2-2\lambda\cos\varphi}} = f(\varphi) \quad (10)$$

So the dimensionless transfer function *f(φ)* is very important for inverted slider-crank mechanism and fully describes its force capabilities. Figure 3 shows the shell diagram of the function *f(φ)*, resp. $M_F(\varphi)$ (case $\lambda = 0.33$).

The function *f(φ)* has an extremum (maximum: $f(\varphi_o) = 1$) in the point $\varphi_o$ (see Figure 2 and Figure 3) determined by:

$$\cos\varphi_o = \lambda \quad (11)$$

### *3.4. Approximation Representation of the Function f(φ)*

The function *f(φ)* has been studied by many authors, looking for different ways to simplify it. In its form (6) the function *f(φ)* is inconvenient for kinematic and dynamic analysis, as it leads to very complex equations [16].

A good method for its approximation representation is given in [17]. Function (6) is developed in binomial order and is taken up to the second power of the small parameter. In the present work we have extended the binomial series to the fourth power of the small parameter *λ*. So the approximation function $f_a(\varphi)$ takes the following form:

$$f_a(\varphi) = \sin\varphi + \frac{1}{2}\lambda\sin 2\varphi + \frac{1}{8}\lambda^2(3\sin 3\varphi - \sin\varphi) +$$
$$+\frac{1}{16}\lambda^3(5\sin 4\varphi - 2\sin 2\varphi) + \quad (12)$$
$$+\frac{1}{128}\lambda^4(35\sin 5\varphi - 15\sin 3\varphi - 2\sin\varphi) + \cdots$$

Figure 4 shows the exact value of the transfer function *f(φ)* and its approximating function $f_a(\varphi)$ taken up to the second power of the small parameter *λ* (the first 3 parts of formula (12)). It can be seen that the function $f_a(\varphi)$ with sufficient accuracy approximates the exact transfer function, with a maximum error of less than 1.6%.

Accordingly, the accuracy of the approximating function increases extremely much when we take it to the third or fourth power of the small parameter *λ* (the first 4 or 5 parts of formula (12)). So the error $f_a(\varphi) - f(\varphi)$, becomes 0.45% and 0.13% respectively.

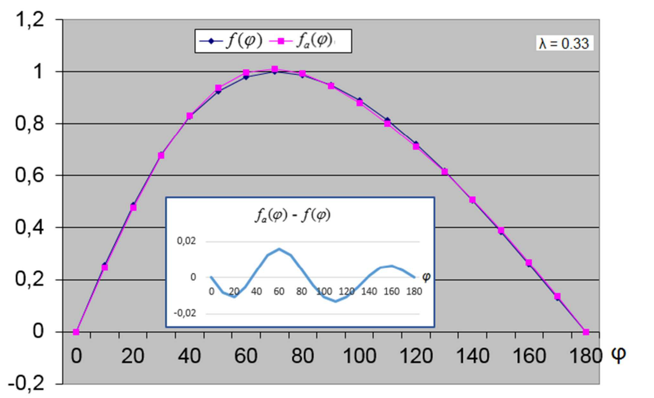


***Figure 4.*** *Transfer functions – exact f(φ) and approximate $f_a(\varphi)$ and their difference ($f_a(\varphi)$ - f(φ)).*

# 4. Transfer Functions of Inverted Slider-Crank Mechanism, with Independent Parameter – the Linear Motion ρ

### *4.1. The Exact Transfer Function*

In the case we accept as independent parameter of inverted slider-crank mechanism, the linear motion of the hydraulic cylinder *ρ*, resp. *s*.

From equations (3) after excluding the angle *θ* we get:

$$\cos\varphi = \frac{L^2+R^2-\rho^2}{2LR} \quad (13)$$

After differentiating (13) by *ρ* we obtain for the transfer function *dφ/dρ* the following one:

$$\frac{d\varphi}{d\rho} = \frac{2\rho}{\sqrt{4L^2R^2-(L^2+R^2-\rho^2)^2}} \quad (14)$$

If the dimensionless parameter *y=ρ/L* is entered, the transfer function takes the form:

$$\frac{d\varphi}{dy} = \frac{2y}{\sqrt{4\lambda^2-(1+\lambda^2-y^2)^2}},\ where\ y = \frac{\rho}{L} \quad (15)$$

The transfer function *dφ/dy* has an extreme (minimum) for the value:

$$y_o = \sqrt{1-\lambda^2} \quad (16)$$

which exactly corresponds to condition (11).

Another change using an independent parameter *z*, can also be made using the relations:

$$\rho = L - R + s$$
$$y = 1 - \lambda + 2\lambda z \quad (17)$$
$$z = \frac{s}{2R} = \frac{s}{s_{max}}$$

The convenience of the parameter *z* is that it varies in the range (0,...,1), which corresponds to the full stroke of the cylinder from zero to its maximum value (*2R*), normalized in dimensionless form.

The transfer function now takes the form:

$$\frac{d\varphi}{dz} = 2.\frac{1-\lambda+2\lambda z}{\sqrt{1-(1-2z(1-\lambda-\lambda z))^2}} \quad (18)$$

### *4.2. Approximation Representation of the Transfer Function dφ/dz*

In this form, the transfer function is quite complex and for the purposes of kinematic and dynamic analysis it is necessary to look for a simpler presentation.

We will look for an approximation representation of the transfer function in the form of a trigonometric function next way:

$$f_a(z) = a_1 + a_2\cos[(z-z_o)\pi] \quad (19)$$

We will set the condition that at the extreme point $z_o$ the exact transfer function $d\varphi/dz$ and its approximating function $f_a(z)$ must be equal to the second derivative:

$$\left(\frac{d\varphi}{dz}\right) = f_a(z)$$
$$\left(\frac{d\varphi}{dz}\right)' = f_a'(z) \quad (20)$$
$$\left(\frac{d\varphi}{dz}\right)'' = f_a''(z)$$

From the conditions (20) we get for the unknowns $z_o$, $a_1$, $a_2$:

$$z_o = \frac{\sqrt{1-\lambda^2}-1+\lambda}{2\lambda}$$
$$a_1 = 2+\frac{2^3}{\pi^2} \quad (21)$$
$$a_2 = -\frac{2^3}{\pi^2}$$

Thus, the approximation transfer function takes the form:

$$f_a(z) = 2+\frac{2^3}{\pi^2}\{1-\cos[(z-z_o)\pi]\} \quad (22)$$

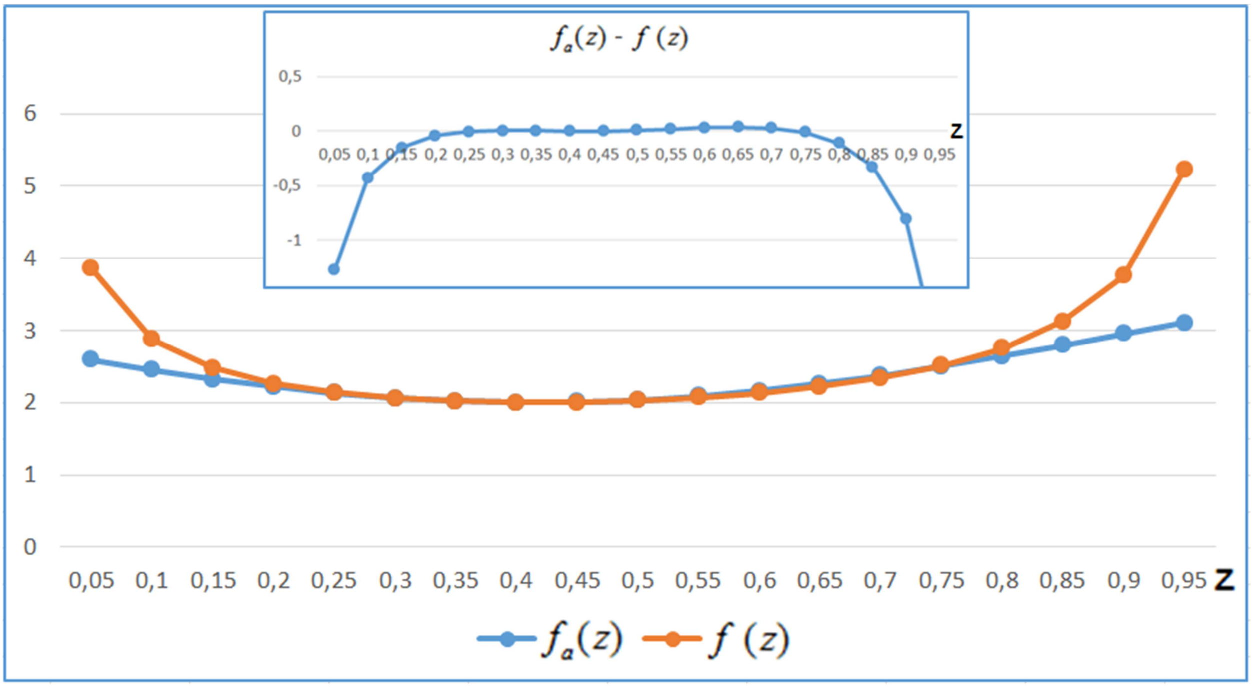


***Figure 5.*** *Transfer functions - exact f(z) and approximate $f_a(z)$ and their difference ($f_a(z)$ - f(z)).*

Figure 5 shows the graphs of the exact *f(z)* and approximate $f_a(z)$ transfer functions (case $\lambda = 0.33$). The good approaching of the approximation function is seen, as in the main operating range of *z* (0.20-0.77) the maximum error does not exceed 2%.

## 5. Conclusion

Kinematic modeling is useful for understanding the behavior and improving the performance of the hydraulic excavators, the main area of application of the mechanism under consideration [18]. The inverted slider-crank mechanism driven by hydraulic cylinder has highly non-linear transfer functions. However, they are subject to approximation representation, which can facilitate their kinematic and dynamic analysis. At the same time, the approximated transfer functions maintain high accuracy in the main operating range. The main transfer function *dρ/dφ* or *f(φ)* is a basic characteristic of these mechanisms. It determines the force capabilities of the mechanism and has a clear geometric interpretation. Thus found transfer functions and their approximations can be used also in the synthesis of this type of mechanisms considering their force capabilities, to achieve certain technological tasks.

## 6. Recommendations

This work builds a detailed mechanical-mathematical model of the inverted slider-crank mechanism driven by a hydraulic cylinder, describing the geometry and transfer functions of the mechanism. The kinematic characteristics of the mechanism at a given movement of the hydraulic cylinder or the rotating pair can be developed. The model constructed in this way can be applied in more complex kinematic chains, including two or three such mechanisms, typical for excavators and some robots, for kinematic and dynamic analysis and for the movement of the end effector according to a certain law. It can also be used successfully in the synthesis of these mechanisms.

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